

Package ‘QuantileGH’

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Type Package

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Description Functions for simulation, estimation, and model selection of finite mixtures of Tukey's g-and-h distributions.

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approxdens *Empirical Density Function*

Description

..

Usage

```
approxdens(x, ...)
```

Arguments

x [numeric vector](#), observations
... additional parameters of [density.default](#)

Details

[approx](#) inside [density.default](#)
another 'layer' of [approxfun](#)

Value

[approxdens](#) returns a [function](#).

Examples

```
x = rnorm(1e3L)  
f = approxdens(x)  
f(x[1:3])
```

as.fmx *Turn Various Objects to fmx*

Description

Turn various objects that are created in other packages to `fmx` class

Usage

```
as.fmx(x, data, ...)
```

Arguments

x	an R object
data	numeric vector
...	..

Details

In order to take advantage of all methods for `fmx` objects

Value

`as.fmx` returns an `fmx` object.

See Also

[as.fmx.fitdist](#) [as.fmx.mixEM](#)

as.fmx.fitdist *Convert fitdist Objects to fmx Objects*

Description

..

Usage

```
## S3 method for class 'fitdist'
as.fmx(x, data = x[["data"]], ...)
```

Arguments

x	fitdist object
data	numeric vector
...	..

Value

[as.fmx.fitted](#) returns an [fmx](#) object.

as.fmx.mixEM	<i>Convert mixEM Objects to fmx Objects</i>
--------------	---

Description

..

Usage

```
## S3 method for class 'mixEM'  
as.fmx(x, data = x[["x"]], ...)
```

Arguments

x	mixEM object
data	numeric vector
...	..

Value

[as.fmx.mixEM](#) returns an [fmx](#) object.

Note

[plot.mixEM](#) not plot [gammamixEM](#) returns, as of 2022-09-19.

Examples

```
library(mixtools)  
(x = as.fmx(normalmixEM(faithful$waiting, k = 2)))
```

as_fmx_mixsmsn

*Convert Objects from **mixsmsn** to **fmx** Objects*

Description

..

Usage

```
## S3 method for class 'Skew.normal'
as.fmx(x, data, engine = c("sn", "mixsmsn"), ...)
```

```
## S3 method for class 'Normal'
as.fmx(x, data, ...)
```

```
## S3 method for class 'Skew.t'
as.fmx(x, data, engine = c("sn", "mixsmsn"), ...)
```

```
## S3 method for class 't'
as.fmx(x, data, ...)
```

Arguments

x	Normal, Skew.normal, t, Skew.t object, returned from smsn.mix with option family = 'Skew.normal'
data	numeric vector
engine	character scalar for Skew.normal and Skew.t method dispatch, either sn (default, using dsn , psn , dst or pst), or mixsmsn (using my re-written dSN and dST , but I do not have function pSN or pST).
...	..

Value

[as.fmx.Skew.normal](#), [as.fmx.Normal](#) and [as.fmx.Skew.t](#) return an [fmx](#) object.

[as.fmx.t](#) has not been completed yet

Note

[smsn.mix](#) does not offer a parameter to keep the input data, as of 2021-10-06.

See Also

[smsn.mix](#)

Examples

```

if (FALSE) {
# all examples work
# disabled as they are slow

library(mixsmsn)
# ?smsn.mix
arg1 = c(mu = 5, sigma2 = 9, lambda = 5, nu = 5)
arg2 = c(mu = 20, sigma2 = 16, lambda = -3, nu = 5)
arg3 = c(mu = 35, sigma2 = 9, lambda = -6, nu = 5)
set.seed(120); x = rmix(n = 1e3L, p=c(.5, .2, .3), family = 'Skew.t',
  arg = list(unname(arg1), unname(arg2), unname(arg3)))

# Skew Normal
class(m <- smsn.mix(x, nu = 3, g = 3, family = 'Skew.normal', calc.im = FALSE))
mix.hist(y = x, model = m)

m2 = as.fmx(m, data = x, engine = 'mixsmsn')
autoplot(m2)
(l2 = logLik(m2))
stopifnot(identical(AIC(m2), m$aic), identical(BIC(m2), m$bic))
tryCatch(fmx_diagnosis(m2, type = 'Kolmogorov'), error = identity)
tryCatch(fmx_diagnosis(m2, type = 'CramerVonMises'), error = identity)

m3 = as.fmx(m, data = x, engine = 'sn')
autoplot(m3)
(l3 = logLik(m3))
stopifnot(identical(AIC(m3), m$aic), identical(BIC(m3), m$bic))
autoplot(m3, type = 'distribution')
fmx_diagnosis(m3, type = 'Kolmogorov')
fmx_diagnosis(m3, type = 'CramerVonMises')

fmx_diagnosis(m2, type = 'KullbackLeibler') - fmx_diagnosis(m3, type = 'KullbackLeibler')

identical(l2, l3)
range(attr(l2, 'logl') - attr(l3, 'logl'))

if (FALSE) {
(x0 = x[729]) # only wrong
#range(qfmx(pfmx(x[-729], dist = m3), dist = m3) - x[-729])
#range(qfmx(pfmx(x[-729], dist = m3, lower.tail = F), dist = m3, lower.tail = F) - x[-729])
qfmx(pfmx(x0, dist = m3), dist = m3)
qfmx(pfmx(x0, dist = m3, lower.tail = FALSE), dist = m3, lower.tail = FALSE)
qfmx_interval(m3) # using interval end. I have no plan to fix this error.
}

# Normal
class(m <- smsn.mix(x, nu = 3, g = 3, family = 'Normal', calc.im = FALSE))
mix.hist(y = x, model = m)
autoplot(as.fmx(m, data = x))

```

```

# skew t
set.seed(131534); x = rmix(n = 1e3L, p = c(.5, .2, .3), family = 'Skew.t',
  arg = list(unnamed(arg1), unnamed(arg2), unnamed(arg3)))

g = 3
class(m <- smsn.mix(x, nu = 3, g = g, family = 'Skew.t', calc.im = FALSE))
mix.hist(y = x, model = m)

m2 = as.fmx(m, data = x, engine = 'mixsmsn')
autoplot(m2)
(l2 = logLik(m2))
attr(l2, 'df') = g*3 + 1 + (g-1) # ?mixsmsn::smsn.mix gives same `nu` for all components
stopifnot(all.equal.numeric(AIC(l2), m$aic), all.equal.numeric(BIC(l2), m$bic))
tryCatch(fmx_diagnosis(m2, type = 'Kolmogorov'), error = identity)
tryCatch(fmx_diagnosis(m2, type = 'CramerVonMises'), error = identity)

m3 = as.fmx(m, data = x, engine = 'sn')
autoplot(m3)
(l3 = logLik(m3))
attr(l3, 'df') = g*3 + 1 + (g-1) # same reason
stopifnot(all.equal.numeric(AIC(l3), m$aic), all.equal.numeric(BIC(l3), m$bic))
autoplot(m3, type = 'distribution')
fmx_diagnosis(m3, type = 'Kolmogorov')
fmx_diagnosis(m3, type = 'CramerVonMises')

fmx_diagnosis(m2, type = 'KullbackLeibler') - fmx_diagnosis(m3, type = 'KullbackLeibler')

identical(l2, l3)
range(attr(l2, 'logl') - attr(l3, 'logl'))

# t
class(m <- smsn.mix(x, nu = 3, g = 3, family = 't', calc.im = FALSE))
mix.hist(y = x, model = m)
# autoplot(as.fmx(m, data = x)) # not ready yet!!
}

```

autolayer_fmx_continuous

*Create **layer** for Continuous **fmx** Objects*

Description

..

Usage

```

autolayer_fmx_continuous(
  object,
  type = c("density", "distribution"),
  data = object@data,
  epdf = object@epdf,
  probs = object@probs,
  xlim = if (!length(data)) qfmx(p = c(0.01, 0.99), dist = object) else
    range.default(data),
  hist.fill = "grey95",
  curve.col = 1,
  n = 1001L,
  ...
)

```

Arguments

object	fmx object
type	character scalar. Option 'density' (default) plots the probability density for fmx input (and the histogram if argument data is available). Option 'distribution' plots the cumulative probability distribution for fmx input (and the empirical cumulative distribution if argument data is available).
data	(optional) numeric vector of the observations. Default is the slot object@data.
epdf	(optional) empirical probability density function returned by approxfun . Default is the slot object@epdf
probs	numeric vector , the percentages (to be) used in QLMDe , can be plotted as vertical lines. Use probs = NULL to suppress the printing of these lines.
xlim	numeric length-two vector , horizontal range
hist.fill	color of the body of histogram, default 'grey95'
curve.col	color of the density curve of the fitted finite mixture distribution. Default 'black'
n	integer , see stat_function
...	potential parameters of stat_function

Value

[autolayer_fmx_continuous](#) returns a [list](#) of [layers](#).

See Also

[autolayer](#) [geom_histogram](#) [stat_function](#)

`autolayer_fmx_discrete`*Create [layer](#) for Discrete [fmx](#) Objects*

Description

..

Usage

```
autolayer_fmx_discrete(  
  object,  
  type = c("density", "distribution"),  
  data = object@data,  
  xlim = if (length(data)) data else qfmx(p = c(0.01, 0.99), dist = object),  
  bins = 60L,  
  ...  
)
```

Arguments

<code>object</code>	fmx object
<code>type</code>	character scalar. Option 'density' (default) plots the probability density for fmx input (and the histogram if argument <code>data</code> is available). Option 'distribution' plots the cumulative probability distribution for fmx input (and the cumulative histogram if argument <code>data</code> is available).
<code>data</code>	(optional) numeric (actually integer) vector of the observations. Default is the slot <code>object@data</code> .
<code>xlim</code>	numeric length-two vector , horizontal range
<code>bins</code>	integer scalar
<code>...</code>	additional parameters, currently not in use

Value

`autolayer_fmx_discrete` returns a [list](#) of [layers](#).

See Also

[autolayer geom_histogram](#)

autoplot.fmx	<i>Plot</i>	<i>fmx</i>	<i>Objects</i>	<i>using</i>	<i>Rhref</i> https://CRAN.R-project.org/package=ggplot2 ggplot2
--------------	-------------	------------	----------------	--------------	---

Description

Plot `fmx` objects using **ggplot2**.

Usage

```
## S3 method for class 'fmx'
autoplot(
  object,
  xlab = attr(object, which = "data.name", exact = TRUE),
  ylab = NULL,
  title = TeX(constraint_TeX(object)),
  caption = NULL,
  ...
)
```

Arguments

`object` `fmx` object
`xlab, ylab, title, caption` `character` scalars, the horizontal and vertical label, title and caption
`...` potential parameters of `autolayer_fmx_continuous` and `autolayer_fmx_discrete`

Value

`autoplot.fmx` returns a `ggplot` object.

See Also

[autoplot labs](#)

Examples

```
(d2 = fmx('GH', A = c(1,6), B = 2, g = c(0,.3), h = c(.2,0), w = c(1,2)))
curve(dfmx(x, dist = d2), xlim = c(-3, 11))
curve(pfmx(x, dist = d2), xlim = c(-3, 11))
autoplot(d2)
autoplot(d2, type = 'distribution')
```

CK5	<i>CK5</i>
-----	------------

Description

..

Usage

CK5

Format

a list of double vectors

clusterList	<i>Clustering Observations for Creation of <code>fmx</code> Object</i>
-------------	--

Description

..

Usage

clusterList(x, K, method = c("reassign_tkmeans"), alpha = 0.05, ...)

Arguments

x	numeric vector, one-dimensional observations
K	integer scalar, number of mixture components
method	character scalar, only 'reassign_tkmeans' supported yet
alpha	numeric scalar, proportion of observations to be trimmed in trimmed k-means algorithm <code>tkmeans</code>
...	additional parameters, currently not in use

Value

clusterList returns a list of numeric vectors.

See Also`kmeans` `tkmeans` `reAssign`

coef.fmx	<i>Parameter Estimates of fmx object</i>
----------	--

Description

..

Usage

```
## S3 method for class 'fmx'
coef(object, internal = FALSE, ...)
```

Arguments

object	fmx object
internal	logical scalar, either for the user-friendly parameters (FALSE, default) (e.g., mean, sd for normal mixture, and A, B, g, h for Tukey's <i>g</i> -and- <i>h</i> mixture), or for the internal/unconstrained parameters (TRUE).
...	place holder for S3 naming convention

Details

[coef.fmx](#) returns the estimates of the user-friendly parameters (`parm = 'user'`), or the internal/unconstrained parameters (`parm = 'internal'`). When the distribution has constraints on one or more parameters, [coef.fmx](#) does not return the estimates (which is constant \emptyset) of the constrained parameters.

Value

[coef.fmx](#) returns a **numeric vector**.

See Also

[coef](#)

confint.fmx	<i>Confidence Interval of fmx Object</i>
-------------	--

Description

...

Usage

```
## S3 method for class 'fmx'
confint(object, ..., level = 0.95)
```

Arguments

object	fmx object
...	place holder for S3 naming convention
level	confidence level, default 95%.

Details

[confint.fmx](#) returns the Wald-type confidence intervals based on the user-friendly parameters (`parm = 'user'`), or the internal/unconstrained parameters (`parm = 'internal'`). When the distribution has constraints on one or more parameters, [confint.fmx](#) does not return the confident intervals of for the constrained parameters.

Value

[confint.fmx](#) returns a [matrix](#)

See Also

[confint](#)

constraint_TeX	<i>TeX Label of Parameter Constraint(s) of fmx Object</i>
----------------	---

Description

Create TeX label of parameter constraint(s) of [fmx](#) object

Usage

```
constraint_TeX(dist)
```

Arguments

dist	fmx object
------	----------------------------

Value

[constraint_TeX](#) returns a [character](#) scalar (of TeX expression) of the constraint, primarily intended for end-users in plots.

Examples

```
(d0 = fmx('GH', A = c(1,4), g = c(.2,.1), h = c(.05,.1), w = c(1,1)))
constraint_TeX(d0)
```

```
(d1 = fmx('GH', A = c(1,4), g = c(.2,0), h = c(0,.1), w = c(1,1)))
constraint_TeX(d1)
```

```
(d2 = fmx('GH', A = c(1,4), g = c(.2,0), h = c(.15,.1), w = c(1,1)))
constraint_TeX(d2)
```

 crossprod_inv

Inverse of $X'X$ by QR Decomposition

Description

Compute $(X'X)^{-1}$ from the R part of the QR decomposition of X .

Usage

```
crossprod_inv(X)
```

Arguments

X *$m * n$ matrix*

Value

`crossprod_inv` returns the inverse [matrix](#) of cross product $X'X$.

References

https://en.wikipedia.org/wiki/QR_decomposition, section **Rectangular matrix**

See Also

[chol2inv](#) [chol.default](#)

[qr.default](#) [qr.R](#) [chol2inv](#)

Examples

```
set.seed(123); (X = array(rnorm(40L), dim = c(8L, 5L)))
stopifnot(all.equal(numeric(solve(crossprod(X))), crossprod_inv(X)))
```

CycD1	<i>CycD1</i>
-------	--------------

Description

..

Usage

CycD1

Formata [list](#) of [double vectors](#)

dbl2fmx	<i>Inverse of fmx2dbl, for internal use</i>
---------	---

Description

..

Usage

dbl2fmx(x, K, distname, ...)

Arguments

x	numeric vector , unrestricted parameters
K	integer scalar
distname	character scalar
...	additional parameters, not currently used

Details

Only used in [QLMDe](#) and unexported function `qfmx_gr`, not compute intensive

Value

`dbl2fmx` returns a [list](#) with two elements 'pars' and 'w'

Description

Density function, distribution function, quantile function and random generation for a finite mixture distribution with normal or Tukey's *g*-&-*h* components.

Usage

```
dfmx(  
  x,  
  dist,  
  distname = dist@distname,  
  K = dim(pars)[1L],  
  pars = dist@pars,  
  w = dist@w,  
  ...,  
  log = FALSE  
)  
  
pfmx(  
  q,  
  dist,  
  distname = dist@distname,  
  K = dim(pars)[1L],  
  pars = dist@pars,  
  w = dist@w,  
  ...,  
  lower.tail = TRUE,  
  log.p = FALSE  
)  
  
qfmx(  
  p,  
  dist,  
  distname = dist@distname,  
  K = dim(pars)[1L],  
  pars = dist@pars,  
  w = dist@w,  
  interval = qfmx_interval(dist = dist),  
  ...,  
  lower.tail = TRUE,  
  log.p = FALSE  
)  
  
rfmx(  
  r,
```

```

    n,
    dist,
    distname = dist@distname,
    K = dim(pars)[1L],
    pars = dist@pars,
    w = dist@w
  )

```

Arguments

`x, q` **numeric vector**, quantiles, `NA_real_` value(s) allowed.

`dist` **fmx** object, a finite mixture distribution

`distname, K, pars, w` auxiliary parameters, whose default values are determined by argument `dist`. The user-specified **vector** of `w` does not need to sum up to 1; $w/\text{sum}(w)$ will be used internally.

`...` additional parameters

`log, log.p` **logical** scalar. If TRUE, probabilities are given as $\log(p)$.

`lower.tail` **logical** scalar. If TRUE (default), probabilities are $Pr(X \leq x)$, otherwise, $Pr(X > x)$.

`p` **numeric vector**, probabilities.

`interval` length two **numeric vector**, interval for root finding, see `vuniroot`

`n` **integer** scalar, number of observations.

Details

A computational challenge in `dfmx` is when mixture density is very close to 0, which happens when the per-component log densities are negative with big absolute values. In such case, we cannot compute the log mixture densities (i.e., `-Inf`), for the log-likelihood using `logLik.fmx`. Our solution is to replace these `-Inf` log mixture densities by the weighted average (using the mixing proportions of `dist`) of the per-component log densities.

`qfmx` gives the quantile function, by numerically solving `pfmx`. One major challenge when dealing with the finite mixture of Tukey's *g*-&-*h* family distribution is that Brent–Dekker's method needs to be performed in both `pGH` and `qfmx` functions, i.e. *two layers* of root-finding algorithm.

Value

`dfmx` returns a **numeric vector** of probability density values of an `fmx` object at specified quantiles `x`.

`pfmx` returns a **numeric vector** of cumulative probability values of an `fmx` object at specified quantiles `q`.

`qfmx` returns an unnamed **numeric vector** of quantiles of an `fmx` object, based on specified cumulative probabilities `p`. Note that `qnorm` returns an unnamed **vector** of quantiles, although `quantile` returns a named **vector** of quantiles.

`rfmx` generates random deviates of an `fmx` object.

Examples

```

x = (-3):7

(e1 = fmx('norm', mean = c(0,3), sd = c(1,1.3), w = c(1, 1)))
isS4(e1) # TRUE
slotNames(e1)
autoplot(e1)
hist(rfmx(n = 1e3L, dist = e1), main = '1000 obs from e1')
# generate a sample of size 1e3L from mixture distribution `e1`
round(dfmx(x, dist = e1), digits = 3L)
round(p1 <- pfmx(x, dist = e1), digits = 3L)
stopifnot(all.equal.numeric(qfmx(p1, dist = e1), x, tol = 1e-4))

(e2 = fmx('GH', A = c(0,3), g = c(.2, .3), h = c(.2, .1), w = c(2, 3)))
hist(rfmx(n = 1e3L, dist = e2), main = '1000 obs from e2')
round(dfmx(x, dist = e2), digits = 3L)
round(p2 <- pfmx(x, dist = e2), digits = 3L)
stopifnot(all.equal.numeric(qfmx(p2, dist = e2), x, tol = 1e-4))

(e3 = fmx('GH', A = 0, g = .2, h = .2)) # one-component Tukey
hist(rfmx(1e3L, dist = e3))
hist(rGH(n = 1e3L, A = 0, g = .2, h = .2))
# identical (up to random seed); but ?rfmx has much cleaner code
round(dfmx(x, dist = e3), digits = 3L)
round(p3 <- pfmx(x, dist = e3), digits = 3L)
stopifnot(all.equal.numeric(qfmx(p3, dist = e3), x, tol = 1e-4))

if (FALSE) {
  # log-mixture-density smoothing, for developers
  (e4 = fmx('norm', mean = c(0,3), w = c(2, 3)))
  curve(dfmx(x, dist = e4, log = TRUE), xlim = c(-50, 50))
}

```

distArgs	<i>Name(s) of Formal Argument(s) of Distribution</i>
----------	--

Description

To obtain the name(s) of distribution parameter(s).

Usage

```
distArgs(distname)
```

Arguments

distname [character](#) scalar, name of distribution

Value

`distArgs` returns a [character vector](#).

See Also

[formalArgs](#)

distType	<i>Distribution Type</i>
----------	--------------------------

Description

..

Usage

```
distType(type = c("discrete", "nonNegContinuous", "continuous"))
```

Arguments

type [character](#) scalar

Value

`distType` returns a [character vector](#).

dist_logtrans	<i>Distribution Parameters that needs to have a log-transformation</i>
---------------	--

Description

..

Usage

```
dist_logtrans(distname)
```

Arguments

distname [character](#) scalar, name of distribution

Value

`dist_logtrans` returns an [integer](#) scalar

 drop1_fmx

Drop or Add One Parameter from [fmx](#) Object

Description

Fit [fmx](#) models with a single parameters being added or dropped.

Usage

```
## S3 method for class 'fmx'
drop1(object, ...)
```

```
## S3 method for class 'fmx'
add1(object, ...)
```

Arguments

```
object      fmx object
...         additional parameters, currently not in use.
```

Details

..

Value

[drop1.fmx](#) and [add1.fmx](#) return a [list](#) of [fmx](#) objects, in the reverse order of model selection.

Note

Note that [drop1.fmx](#) and [add1.fmx](#) do **not** return an [anova](#) table, like other `stats:::drop.*` or `stats:::add1.*` functions do.

See Also

[step drop1 add1](#)

Examples

```
# donttest to save time

(d2 = fmx('GH', A = c(1,6), B = 1.2, g = c(0,.3), h = c(.2,0), w = c(1,2)))
set.seed(3123); hist(x2 <- rfm(x2, n = 1e3L, dist = d2))
system.time(m0 <- QLMDe(x2, distname = 'GH', K = 2L, constraint = c('g1', 'g2', 'h1', 'h2')))
system.time(m1 <- QLMDe(x2, distname = 'GH', K = 2L, constraint = c('g1', 'h2')))
```

```

system.time(m2 <- QLMD(x2, distname = 'GH', K = 2L)) # ~2 secs

d1 = drop1(m1)
d1 # NULL
d2 = drop1(m2)
vapply(d2, FUN = constraint_TeX, FUN.VALUE = '')

a0 = add1(m0)
vapply(a0, FUN = constraint_TeX, FUN.VALUE = '')
a1 = add1(m1)
vapply(a1, FUN = constraint_TeX, FUN.VALUE = '')

```

fmx

Create Finite Mixture Distribution

Description

..

Usage

```
fmx(distname, w = 1, ...)
```

Arguments

distname	character scalar
w	(optional) numeric vector. Does not need to sum up to 1; w/sum(w) will be used internally.
...	mixture distribution parameters. See dGH for the names and default values of Tukey's <i>g</i> -&- <i>h</i> distribution parameters, or dnorm for the names and default values of normal distribution parameters.

Value

[fmx](#) returns an [fmx](#) object which specifies the parameters of a finite mixture distribution.

Examples

```

(e1 = fmx('norm', mean = c(0,3), sd = c(1,1.3), w = c(1, 1)))
isS4(e1) # TRUE
slotNames(e1)

(e2 = fmx('GH', A = c(0,3), g = c(.2, .3), h = c(.2, .1), w = c(2, 3)))

(e3 = fmx('GH', A = 0, g = .2, h = .2)) # one-component Tukey

```

fmx-class

*Specification of fmx Class***Description**

Parameters and type of distribution of a one-dimensional finite mixture.

Slots

distname **character** scalar, name of parametric distribution of the mixture components. Currently, normal ('norm') and Tukey's *g*-&-*h* ('GH') distributions are supported.

pars **double matrix**, all distribution parameters in the mixture. Each row corresponds to one component. Each column includes the same parameters of all components. The order of rows corresponds to the (non-strictly) increasing order of the component location parameters. The columns match the formal arguments of the corresponding distribution, e.g., 'mean' and 'sd' for **normal** mixture, or 'A', 'B', 'g' and 'h' for Tukey's *g*-&-*h* mixture.

w **numeric vector** of mixing proportions that must sum to 1

data (optional) **numeric vector**, the one-dimensional observations

data.name (optional) **character** scalar, a human-friendly name of observations

epdf (optional) empirical probability density **function** returned by **approxfun**

vcov_internal (optional) variance-covariance **matrix** of the internal (i.e., unconstrained) estimates

vcov (optional) variance-covariance **matrix** of the mixture distribution (i.e., constrained) estimates

probs (optional) **numeric** vectors of probabilities, where the **quantiles** could be calculated

fmx2dbl

*Reparameterization of fmx Object***Description**

To convert the parameters of **fmx** object into unrestricted parameters.

Usage

```
fmx2dbl(
  x,
  distname = x@distname,
  pars = x@pars,
  K = dim(pars)[1L],
  w = x@w,
  ...
)
```

Arguments

x	fmx object
distname	character scalar, default value from x@distname
pars	numeric matrix , default value from x@pars
K	integer scalar, default value from x
w	numeric vector , default value from x@w
...	additional parameters, not currently used

Details

For the first parameter

- $A_1 \rightarrow A_1$
- $A_2 \rightarrow A_1 + \exp(\log(d_1))$
- $A_k \rightarrow A_1 + \exp(\log(d_1)) + \dots + \exp(\log(d_{k-1}))$

For mixing proportions to multinomial logits.

For 'norm': sd -> log(sd) for 'GH': B -> log(B), h -> log(h)

Value

[fmx2dbl](#) returns a [numeric](#) vector

See Also

[dbl2fmx](#)

fmx_cluster

Naive Estimates of Finite Mixture Distribution via Clustering

Description

Naive estimates for finite mixture distribution [fmx](#) via clustering.

Usage

```
fmx_cluster(
  x,
  K,
  distname = c("GH", "norm", "sn"),
  constraint = character(),
  ...
)
```


Arguments

x	numeric vector, observations
K	integer scalar, number of mixture components
distname	character scalar, name of parametric distribution of the mixture components
constraint	character vector, parameters (g and/or h for Tukey's g -&- h mixture) to be set at 0. See <code>fmx_constraint</code> for details.
...	additional parameters, currently not in use

Details

First of all, if the specified number of components $K \geq 2$, trimmed k -means clustering with re-assignment will be performed; otherwise, all observations will be considered as one single cluster. The standard k -means clustering is not used since the heavy tails of Tukey's g -&- h distribution could be mistakenly classified as individual cluster(s).

In each of the one or more clusters,

- The letter-value based estimates of Tukey's g -&- h distribution (Hoaglin, 2006) are calculated, for any $K \geq 1$, serving as the starting values for QLMD algorithm. These estimates are provided by `fmx_cluster`.
- the `median` and `MAD` will serve as the starting values for μ and σ (or A and B for Tukey's g -&- h distribution, with $g = h = 0$), for QLMD algorithm when $K = 1$.

Value

`fmx_cluster` returns an `fmx` object.

See Also

`letterValue`

fmx_constraint	<i>Parameter Constraint(s) of Mixture Distribution</i>
----------------	--

Description

Determine the parameter constraint(s) of a finite mixture distribution `fmx`, either by the value of parameters of such mixture distribution, or by a user-specified string.

Usage

```
fmx_constraint(
  dist,
  distname = dist@distname,
  K = dim(dist@pars)[1L],
  pars = dist@pars
)
```

Arguments

dist	(optional) fmx object
distname	character scalar, name of distribution (see fmx), default value determined by dist
K	integer scalar, number of components, default value determined by dist
pars	double matrix , distribution parameters of a finite mixture distribution (see fmx), default value determined by dist

Value

fmx_constraint returns the indexes of internal parameters (only applicable to Tukey's *g*-&-*h* mixture distribution, yet) to be constrained, based on the input **fmx** object dist.

Examples

```
(d0 = fmx('GH', A = c(1,4), g = c(.2,.1), h = c(.05,.1), w = c(1,1)))
(c0 = fmx_constraint(d0))
stopifnot(identical(c0, user_constraint(character(), distname = 'GH', K = 2L)))

(d1 = fmx('GH', A = c(1,4), g = c(.2,0), h = c(0,.1), w = c(1,1)))
(c1 = fmx_constraint(d1))
stopifnot(identical(c1, user_constraint(c('g2', 'h1'), distname = 'GH', K = 2L)))

(d2 = fmx('GH', A = c(1,4), g = c(.2,0), h = c(.15,.1), w = c(1,1)))
(c2 = fmx_constraint(d2))
stopifnot(identical(c2, user_constraint('g2', distname = 'GH', K = 2L)))
```

fmx_diagnosis

Diagnoses for fmx Estimates

Description

Diagnoses for **fmx** estimates.

Usage

```
fmx_diagnosis(
  x,
  data = x@data,
  type = c("Kolmogorov", "KullbackLeibler", "CramerVonMises"),
  nullname = deparse1(substitute(x)),
  ...
)
```

Arguments

x	fmx object, or an R object convertible to an fmx object
data	double vector , the actual observations, default value is @data
type	character scalar, currently supports 'Kolmogorov' distance (default), 'KullbackLeibler' divergence, and 'CramerVonMises' test of goodness-of-fit
nullname	character scalar, see cvm.test
...	additional parameters, currently not in use

Value

[fmx_diagnosis](#) returns either

- a [numeric](#) scalar of 'Kolmogorov' distance;
- a [list](#) when type = 'KullbackLeibler', which is returned from LaplacesDemon::KLD.
- an [htest](#) object when type = 'CramerVonMises', in which the element \$statistic is the Cramer-Von Mises quadratic distance.

Note

[cvm.test](#)

See Also

LaplacesDemon::KLD [cvm.test](#) [ks.test](#)

fmx_hybrid

Best Naive Estimates for Finite Mixture Distribution

Description

Best estimates for finite mixture distribution [fmx](#).

Usage

```
fmx_hybrid(x, test = c("logLik", "CvM", "KS"), ...)
```

Arguments

x	numeric vector , observations
test	character scalar, criteria for selecting the optimal estimates. See Details .
...	additional parameters of fmx_normix and fmx_cluster

Details

`fmx_hybrid` compares the Tukey's *g*-&-*h* mixture estimate provided by `fmx_cluster` and the normal mixture estimate by `fmx_normix`, and select the one either with maximum likelihood (`test = 'logLik'`, default), with minimum Cramer-von Mises distance (`test = 'CvM'`) or with minimum Kolmogorov distance (`test = 'Kolmogorov'`).

Value

`fmx_hybrid` returns an `fmx` object.

Examples

```
d1 = fmx('norm', mean = c(1, 2), sd = .5, w = c(.4, .6))
set.seed(100); hist(x1 <- rfm(x1 = 1e3L, dist = d1))
fmx_normix(x1, distname = 'norm', K = 2L)
fmx_normix(x1, distname = 'GH', K = 2L)

(d2 = fmx('GH', A = c(1,6), B = 2, g = c(0,.3), h = c(.2,0), w = c(1,2)))
set.seed(100); hist(x2 <- rfm(x2 = 1e3L, dist = d2))
fmx_cluster(x2, K = 2L)
fmx_cluster(x2, K = 2L, constraint = c('g1', 'h2'))
fmx_normix(x2, K = 2L, distname = 'GH')
fmx_hybrid(x2, distname = 'GH', K = 2L)
```

fmx_normix

Naive Estimates of Finite Mixture Distribution using Mixture of Normal

Description

Naive estimates for finite mixture distribution `fmx` using mixture of normal

Usage

```
fmx_normix(x, K, distname = c("GH", "norm", "sn"), alpha = 0.05, R = 10L, ...)
```

Arguments

<code>x</code>	numeric vector , observations
<code>K</code>	integer scalar, number of mixture components
<code>distname</code>	character scalar, name of parametric distribution of the mixture components
<code>alpha</code>	numeric scalar, proportion of observations to be trimmed in trimmed k-means algorithm <code>tkmeans</code>
<code>R</code>	integer scalar, number of <code>normalmixEM</code> replicates
<code>...</code>	additional parameters, currently not in use

Details

[fmx_normix](#) ... the cluster centers are provided as the starting values of μ 's for the univariate normal mixture by EM [algorithm](#). R replicates of normal mixture estimates are obtained, and the one with maximum likelihood will be selected

Value

[fmx_normix](#) returns an [fmx](#) object.

See Also

[tkmeans](#) [normalmixEM](#)

Kolmogorov_dist	<i>One-Sample Kolmogorov Distance</i>
-----------------	---------------------------------------

Description

To calculate the one-sample Kolmogorov distance between observations and a distribution.

Usage

```
Kolmogorov_dist(
  obs,
  null,
  alternative = c("two.sided", "less", "greater"),
  ...
)
```

Arguments

obs	numeric or integer vector, observations
null	cumulative distribution function
alternative	character scalar, alternative hypothesis, either 'two.sided' (default), 'less', or 'greater'
...	additional arguments of null

Details

[Kolmogorov_dist](#) is different from [ks.test](#) in the following aspects

- Ties in observations are supported. The step function of empirical distribution is inspired by [ecdf](#). This is superior than $(0:(n - 1))/n$ of `stats::ks.test.default`.
- Discrete distribution (with discrete observation) is supported.
- Discrete distribution (with continuous observation) is not supported yet. This will be an easy modification in future.
- Only the one-sample Kolmogorov distance, not the one-sample Kolmogorov test, is returned, to speed up the calculation.

Value

`Kolmogorov_dist` returns a [numeric](#) scalar.

See Also

[ks.test](#)

Examples

```
# from ?stats::ks.test
x1 <- rnorm(50)
ks.test(x1+2, y = pgamma, shape = 3, rate = 2)
Kolmogorov_dist(x1+2, null = pgamma, shape = 3, rate = 2) # exactly the same

# discrete distribution
x2 <- rbinom(n = 1e2L, size = 500, prob = .4)
suppressWarnings(ks.test(x2, y = pbinom, size = 500, prob = .4)) # warning on ties
Kolmogorov_dist(x2, null = pbinom, size = 500, prob = .4) # wont be the same
```

letterValue

Letter-Value Estimation of Tukey g- & h Distribution

Description

Letter-value based estimation (Hoaglin, 2006) of Tukey's *g*-, *h*- and *g*-&-*h* distribution. All equation numbers mentioned below refer to Hoaglin (2006).

Usage

```
letterValue(
  x,
  p_g = seq.int(from = 0.15, to = 0.25, by = 0.005),
  p_h = seq.int(from = 0.15, to = 0.35, by = 0.005),
  halfSpread = c("both", "lower", "upper"),
  ...
)

letterV_B_g_h(A, g, p_h, x, halfSpread, ...)

letterV_B_h(A, p_h, x, halfSpread)

letterV_B(A, g, p_g, x, halfSpread)

letterV_g(A, p_g, x)
```

Arguments

x	double vector , one-dimensional observations
p_g	double vector , probabilities used for estimating g parameter. Or, use p_g = FALSE to implement the constraint $g = 0$ (i.e., an h -distribution is estimated).
p_h	double vector , probabilities used for estimating h parameter. Or, use p_h = FALSE to implement the constraint $h = 0$ (i.e., a g -distribution is estimated).
halfSpread	character scalar, either to use 'both' half-spreads (default), 'lower' half-spread, or 'upper' half-spread.
...	additional parameters, currently not in use
A, g	estimated location \hat{A} and skewness \hat{g}

Details

letterV_g estimates parameter g using equation (10) for g -distribution and the equivalent equation (31) for g -&- h distribution.

letterV_B estimates parameter B for Tukey's g -distribution (i.e., $g \neq 0, h = 0$), using equation (8a) and (8b).

letterV_B_g_h estimates parameters B and h when $g \neq 0$, using equation (33).

letterV_B_h estimates parameters B and h for Tukey's h -distribution, i.e., when $g = 0$ and $h \neq 0$, using equation (26a), (26b) and (27).

letterValue plays a similar role as `fitdistrplus:::start.arg.default`, thus extends **fitdist** for estimating Tukey's g -&- h distributions.

Value

letterValue returns a **double vector** of estimates $(\hat{A}, \hat{B}, \hat{g}, \hat{h})$ for a Tukey's g -&- h distribution.

References

Hoaglin, D.C. (2006). Summarizing Shape Numerically: The g -and- h Distributions. In *Exploring Data Tables, Trends, and Shapes* (eds D.C. Hoaglin, F. Mosteller and J.W. Tukey), Wiley Series in Probability and Statistics. doi:10.1002/9781118150702.ch11

See Also

fitdist

Examples

```
set.seed(77652); x = rGH(n = 1e3L, g = -.3, h = .1)
letterValue(x, p_g = FALSE, p_h = FALSE)
letterValue(x, p_g = FALSE)
letterValue(x, p_h = FALSE)

(y0 = letterValue(x))
library(fitdistrplus)
fit = fitdist(x, distr = 'GH', start = as.list.default(y0))
```

```
plot(fit) # fitdistrplus:::plot.fitdist
```

logLik.fitdist	<i>Log-Likelihood of fitdist Object</i>
----------------	---

Description

..

Usage

```
## S3 method for class 'fitdist'  
logLik(object, ...)
```

Arguments

object	fitdist object
...	additional parameters, currently not in use

Details

Output of [fitdist](#) has element `$loglik` (as well as `$aic` and `$bic`), but it is simply a [numeric](#) scalar. `fitdistrplus:::logLik.fitdist` simply returns this element.

`logLik.fitdist` returns a [logLik](#) object, which could be further used by [AIC](#) and [BIC](#).

(I have written to the authors)

Value

`logLik.fitdist` returns a [logLik](#) object

See Also

[fitdist](#)

logLik.fmx	<i>Log-Likelihood of fmx Object</i>
------------	---

Description

..

Usage

```
## S3 method for class 'fmx'
logLik(object, data = object@data, ...)
```

Arguments

object	fmx object
data	double vector , actual observations
...	place holder for S3 naming convention

Details

[logLik.fmx](#) returns a [logLik](#) object indicating the log-likelihood. An additional attribute `attr(, 'log1')` indicates the point-wise log-likelihood, to be use in Vuong's closeness test.

Value

[logLik.fmx](#) returns a [logLik](#) object with an additional attribute `attr(, 'log1')`.

See Also

[logLik](#)

logLik.mixEM	<i>Log-Likelihood of 'mixEM' Object</i>
--------------	---

Description

To obtain the log-Likelihood of 'mixEM' object, based on [mixtools](#) 2020-02-05.

Usage

```
## S3 method for class 'mixEM'
logLik(object, ...)
```

Arguments

object 'mixEM' object, currently only the returned value of [normalmixEM](#) and [gam-mamixEM](#) are supported

... additional parameters, currently not in use

Value

[logLik.mixEM](#) returns a [logLik](#) object.

See Also

[logLik](#)

mahalanobis_int *A Simpler and Faster Mahalanobis Distance*

Description

A simpler and faster [Mahalanobis](#) distance.

Usage

```
mahalanobis_int(x, center, invcov)
```

Arguments

x [numeric vector](#)

center [numeric vector](#), mean μ

invcov [numeric matrix](#), *inverted* variance-covariance Σ

Value

[mahalanobis_int](#) returns a [numeric](#) scalar.

See Also

[mahalanobis](#)

mixEM_pars	<i>Names of Distribution Parameters of mixEM Object</i>
------------	---

Description

Names of distribution parameters of 'mixEM' object, based on **mixtools** 2020-02-05.

Usage

```
mixEM_pars(object)
```

Arguments

object 'mixEM' object, currently only the returned value of [normalmixEM](#) and [gammamixEM](#) are supported

Value

[mixEM_pars](#) returns a [character](#) vector

See Also

[normalmixEM](#) [gammamixEM](#)

mixsmsn_skewNormal	<i>Skew Normal as in</i>	Rhrefhttps://CRAN.R-project.org/package=mixsmsn mixsmsn
--------------------	--------------------------	---

Description

..

Usage

```
dSN(x, mean = 0, sd = 1, shape = 0, log = FALSE)
```

```
pSN(q, mean = 0, sd = 1, shape = 0, lower.tail = TRUE, log.p = FALSE)
```

Arguments

x, q	..
mean	..
sd	..
shape	..
log, log.p	..
lower.tail	..

Value

`dSN` returns a [numeric vector](#) of the same length as `x`, or a [numeric matrix](#) of the same dimension as `x`.

See Also

`mixsmsn:::dSN` [dsn](#)

mixsmsn_skewT	<i>Skew t as in</i>	Rhref https://CRAN.R-project.org/package=mixsmsn mixsmsn
---------------	---------------------	--

Description

..

Usage

`dST(x, mean = 0, sd = 1, shape = 0, nu = 4, log = FALSE)`

`pST(q, mean = 0, sd = 1, shape = 0, nu = 4, lower.tail = TRUE, log.p = FALSE)`

Arguments

<code>x, q</code>	..
<code>mean</code>	..
<code>sd</code>	..
<code>shape</code>	..
<code>nu</code>	..
<code>log, log.p</code>	..
<code>lower.tail</code>	..

Value

`dST` returns a [numeric vector](#) of the same length as `x`, or a [numeric matrix](#) of the same dimension as `x`.

See Also

`mixsmsn:::dt.ls` [dst](#)

Description

Performs transformation between [vectors](#) of multinomial probabilities and multinomial logits.

This transformation is a generalization of [plogis](#) which converts scalar logit into probability and [qlogis](#) which converts probability into scalar logit.

Usage

```
qmlogis_first(p)
```

```
qmlogis_last(p)
```

```
pmlogis_first(q)
```

```
pmlogis_last(q)
```

Arguments

`p` [numeric vector](#), multinomial probabilities, adding up to 1

`q` [numeric vector](#), multinomial logits

Details

[pmlogis_first](#) and [pmlogis_last](#) take a length $k-1$ [numeric vector](#) of multinomial logits q and convert them into length k multinomial probabilities p , regarding the first or last category as reference, respectively.

[qmlogis_first](#) and [qmlogis_last](#) take a length k [numeric vector](#) of multinomial probabilities p and convert them into length $k-1$ multinomial logits q , regarding the first or last category as reference, respectively.

Value

[pmlogis_first](#) and [pmlogis_last](#) return a [vector](#) of multinomial probabilities p .

[qmlogis_first](#) and [qmlogis_last](#) returns a [vector](#) of multinomial logits q .

See Also

[plogis](#) [qlogis](#)

Examples

```

(a = qmlogis_last(c(2,5,3)))
(b = qmlogis_first(c(2,5,3)))
pmlogis_last(a)
pmlogis_first(b)

q0 = .8300964
(p1 = pmlogis_last(q0))
(q1 = qmlogis_last(p1))

# various exceptions
pmlogis_first(qmlogis_first(c(1, 0)))
pmlogis_first(qmlogis_first(c(0, 1)))
pmlogis_first(qmlogis_first(c(0, 0, 1)))
pmlogis_first(qmlogis_first(c(0, 1, 0, 0)))
pmlogis_first(qmlogis_first(c(1, 0, 0, 0)))
pmlogis_last(qmlogis_last(c(1, 0)))
pmlogis_last(qmlogis_last(c(0, 1)))
pmlogis_last(qmlogis_last(c(0, 0, 1)))
pmlogis_last(qmlogis_last(c(0, 1, 0, 0)))
pmlogis_last(qmlogis_last(c(1, 0, 0, 0)))

```

nobs.fitdist

Number of Observations in [fitdist](#) Object

Description

..

Usage

```

## S3 method for class 'fitdist'
nobs(object, ...)

```

Arguments

```

object      fitdist object
...         additional parameters, currently not in use

```

Value

[nobs.fitdist](#) returns an [integer](#) scalar

See Also

[fitdist](#)

nobs.fmx	<i>Number of Observations in fmx Object</i>
----------	---

Description

..

Usage

```
## S3 method for class 'fmx'  
nobs(object, ...)
```

Arguments

object	fmx object
...	place holder for S3 naming convention

Details

[nobs.fmx](#) returns the sample size of the observations used in [QLMDe](#) estimation, or `integer(0)` for distribution-only [fmx](#) object

Value

[nobs.fmx](#) returns an [integer](#) scalar.

See Also

[nobs](#)

npar.fmx	<i>Number of Parameters of fmx Object</i>
----------	---

Description

..

Usage

```
npar.fmx(dist)
```

Arguments

dist	fmx object
------	----------------------------

Details

Also the degree-of-freedom in [logLik](#), as well as `stats:::AIC.logLik` and `stats:::BIC.logLik`

Value

`npar.fmx` returns an [integer](#) scalar.

`outer_allequal` *Test if Two [double](#) Vectors are Element-Wise (Nearly) Equal*

Description

Test if two [double](#) vectors are element-wise (nearly) equal.

Usage

```
outer_allequal(target, current, tolerance = sqrt(.Machine$double.eps), ...)
```

Arguments

<code>target</code>	length- n double vector , the target value, missing value not allowed
<code>current</code>	length- n double vector , the value to be compared with <code>target</code> , missing value not allowed
<code>tolerance</code>	positive double scalar, default <code>sqrt(.Machine\$double.eps)</code>
<code>...</code>	potential parameters, currently not in use

Details

`outer_allequal` is different from [all.equal.numeric](#), such that (1). only comparisons between real [double](#) values are performed; (2). element-wise comparison is performed, with the rows of returned [matrix](#) correspond to `current` and columns correspond to `target`; (3). a [logical](#) scalar is always returned for each element-wise comparison.

Value

`outer_allequal` returns an $m * n$ [logical matrix](#) indicating whether the length- n [vector](#) `current` is element-wise near-equal to the length- m [vector](#) `target` within the pre specified tolerance.

See Also

[all.equal.numeric](#) [outer](#)

Examples

```
x = c(.3, 1-.7, 0, .Machine$double.eps)
outer_allequal(current = x, target = c(.3, 0))
```

print.fmx	<i>S3 print of fmx Object</i>
-----------	-------------------------------

Description

..

Usage

```
## S3 method for class 'fmx'
print(x, ...)
```

Arguments

x	an fmx object
...	additional parameters, not currently in use

Value

[print.fmx](#) returns the input [fmx](#) object invisibly.

See Also

[print](#)

QLMDe	<i>Quantile Least Mahalanobis Distance estimates</i>
-------	--

Description

The quantile least Mahalanobis distance algorithm estimates the parameters of single-component or finite mixture distributions by minimizing the Mahalanobis distance between the vectors of sample and theoretical quantiles. See [QLMDp](#) for the default selection of probabilities at which the sample and theoretical quantiles are compared.

The default initial values are estimated based on trimmed k -means clustering with re-assignment.

Usage

```
QLMDe(
  x,
  distname = c("GH", "norm", "sn"),
  K,
  data.name = deparse1(substitute(x)),
  constraint = character(),
  probs = QLMDp(x = x),
```

```

init = c("logLik", "letterValue", "normix"),
tol = .Machine$double.eps^0.25,
maxiter = 1000,
...
)

```

Arguments

<code>x</code>	numeric vector , the one-dimensional observations.
<code>distname</code>	character scalar, name of mixture distribution to be fitted. Currently supports 'norm' and 'GH'.
<code>K</code>	integer scalar, number of components (e.g., must use 2L instead of 2).
<code>data.name</code>	character scalar, name for the observations for user-friendly print out.
<code>constraint</code>	character vector , parameters (g and/or h for Tukey's g -&- h mixture) to be set at 0. See fmx_constraint for details.
<code>probs</code>	numeric vector , percentiles at where the sample and theoretical quantiles are to be matched. See QLMDp for details.
<code>init</code>	character scalar for the method of initial values selection, or an fmx object of the initial values. See fmx_hybrid for more details.
<code>tol, maxiter</code>	see vuniroot2
<code>...</code>	additional parameters of optim

Details

Quantile Least Mahalanobis Distance estimator fits a single-component or finite mixture distribution by minimizing the Mahalanobis distance between the theoretical and observed quantiles, using the empirical quantile variance-covariance matrix [quantile_vcov](#).

Value

[QLMDe](#) returns an [fmx](#) object.

See Also

[optim](#) [fmx_hybrid](#)

Examples

```

hist(x1 <- CK5[[1L]])
QLMDe(x1, distname = 'GH', K = 2L)

```

QLMDe_stepK

*Forward Selection of the Number of Components K***Description**

To compare gh -parsimonious models of Tukey's g -&- h mixtures with different number of components K (up to a user-specified K_{\max}) and select the optimal number of components.

Usage

```
QLMDe_stepK(
  x,
  distname = c("GH", "norm"),
  data.name = deparse1(substitute(x)),
  Kmax = 3L,
  test = c("BIC", "AIC"),
  direction = c("forward", "backward"),
  ...
)
```

Arguments

<code>x</code>	numeric vector , observations
<code>distname</code>	character scalar, see QLMDe
<code>data.name</code>	character scalar, see QLMDe
<code>Kmax</code>	integer scalar K_{\max} , maximum number of components to be considered. Default 3L
<code>test</code>	character scalar, criterion to be used, either Akaike's information criterion AIC , or Bayesian information criterion BIC (default).
<code>direction</code>	character scalar, direct of selection in step_fmx , either 'forward' (default) or 'backward'
<code>...</code>	additional parameters

Details

[QLMDe_stepK](#) compares the gh -parsimonious models with different number of components K , and selects the optimal number of components using [BIC](#) (default) or [AIC](#).

The forward selection starts with finding the gh -parsimonious model (via [step_fmx](#)) at $K = 1$. Let the current number of component be K^c . We compare the gh -parsimonious models of $K^c + 1$ and K^c component, respectively, using [BIC](#) or [AIC](#). If K^c is preferred, then the forward selection is stopped, and K^c is considered the optimal number of components. If $K^c + 1$ is preferred, then the forward selection is stopped if $K^c + 1 = K_{\max}$, otherwise update K^c with $K_c + 1$ and repeat the previous steps.

Value

`QLMDe_stepK` returns an object of S3 class 'stepK', which is a [list](#) of selected models (in reversed order) with attribute(s) 'direction' and 'test'.

Examples

```
hist(x1 <- CK5[[1L]])
QLMDe_stepK(x1, distname = 'GH', Kmax = 2L)
```

QLMDp

Percentages for Quantile Least Mahalanobis Distance estimation

Description

A vector of probabilities to be used in Quantile Least Mahalanobis Distance estimation ([QLMDe](#)).

Usage

```
QLMDp(
  from = 0.05,
  to = 0.95,
  length.out = 15L,
  equidistant = c("prob", "quantile"),
  extra = c(0.005, 0.01, 0.02, 0.03, 0.97, 0.98, 0.99, 0.995),
  x
)
```

Arguments

from	numeric scalar, minimum of the equidistant (in probability or quantile) probabilities. Default .05.
to	numeric scalar, maximum of the equidistant (in probability or quantile) probabilities. Default .95.
length.out	non-negative integer scalar, the number of the equidistant (in probability or quantile) probabilities.
equidistant	character scalar. If 'prob' (default), then the probabilities are equidistant. If 'quantile', then the quantiles (of the observations x) corresponding to the probabilities are equidistant.
extra	numeric vector of <i>additional</i> probabilities, default <code>c(.005, .01, .02, .03, .97, .98, .99, .995)</code> .
x	numeric vector of observations, only used when <code>equidistant = 'quantile'</code> .

Details

The default arguments of `QLMDp` returns the probabilities of `c(.005, .01, .02, .03, seq.int(.05, .95, length.out = 15L), .97, .98, .99, .995)`.

Value

A **numeric vector** of probabilities to be supplied to parameter `p` of Quantile Least Mahalanobis Distance `QLMDe` estimation). In practice, the length of this probability **vector** `p` must be equal or larger than the number of parameters in the distribution model to be estimated.

Examples

```
(d2 = fmx('GH', A = c(1,6), B = 2, g = c(0,.3), h = c(.2,0), w = c(1,2)))
set.seed(100); hist(x2 <- rfm(x = 1e3L, dist = d2))
p_hist = geom_histogram(
  mapping = aes(x = x2, y = after_stat(density)), bins = 30L, colour = 'white', alpha = .1)

(p1 = QLMDp()) # equidistant in probabilities
autoplot(d2, probs = p1) + p_hist

(p2 = QLMDp(equidistant = 'quantile', x = x2)) # equidistant in quantiles
autoplot(d2, probs = p2) + p_hist
```

 quantile_vcov

Variance-Covariance of Quantiles

Description

Computes the variance-covariance matrix of quantiles based on Theorem 1 and 2 of Mosteller (1946).

Usage

```
quantile_vcov(probs, d)
```

Arguments

`probs` **numeric vector**, cumulative probabilities at the given quantiles
`d` **numeric vector**, probability densities at the given quantiles

Details

The end user should make sure no density too close to 0 is included in argument `d`. `quantile_vcov` must not be used in a compute-intensive way.

Value

`quantile_vcov` returns the variance-covariance [matrix](#) of quantiles based on Mosteller (1946).

References

Frederick Mosteller. On Some Useful "Inefficient" Statistics. *The Annals of Mathematical Statistics*, 17 (4) 377-408, December, 1946. [doi:10.1214/aoms/1177730881](https://doi.org/10.1214/aoms/1177730881)

reAssign	<i>Re-Assign Observations Trimmed Prior to Trimmed k-Means Clustering</i>
----------	---

Description

Re-assign the observations, which are trimmed in the trimmed k -means algorithm, back to the closest cluster as determined by the smallest Mahalanobis distance.

Usage

```
reAssign(x, ...)  
  
## S3 method for class 'tkmeans'  
reAssign(x, ...)
```

Arguments

<code>x</code>	a tkmeans object
<code>...</code>	potential parameters, currently not in use.

Details

Given the [tkmeans](#) input, the [Mahalanobis](#) distance is computed between each trimmed observation and each cluster. Each trimmed observation is assigned to the closest cluster (i.e., with the smallest Mahalanobis distance).

Value

An 'reAssign_tkmeans' object, which inherits from [tkmeans](#) class.

Note

Either [kmeans](#) or [tkmeans](#) is slow for big x .

See Also

[tkmeans](#)

Examples

```
library(tclust)
data(geyser2)
clus = tkmeans(geyser2, k = 3L, alpha = .03)
plot(clus, main = 'Before Re-Assigning')
plot(reAssign(clus), main = 'After Re-Assigning')
```

show, fmx-method	<i>Show fmx Object</i>
------------------	------------------------

Description

Print the parameters of an `fmx` object and plot its density curves.

Usage

```
## S4 method for signature 'fmx'
show(object)
```

Arguments

`object` an `fmx` object

Value

The `show` method for `fmx` object does not have a returned value.

sort.mixEM	<i>Sort mixEM Object by First Parameters</i>
------------	--

Description

To sort a `mixEM` object by its first parameters, i.e., μ 's for normal mixture, α 's for γ -mixture, etc.

Usage

```
## S3 method for class 'mixEM'
sort(x, decreasing = FALSE, ...)
```

Arguments

`x` `mixEM` object

`decreasing` **logical** scalar. Should the sort by μ 's be increasing (FALSE, default) or decreasing (TRUE)?

`...` additional parameters, currently not in use

Details

[normalmixEM](#) does **not** order the location parameter

Value

[sort.mixEM](#) returns a mixEM object.

See Also

[sort](#)

sort_mixsmsn

*Sort Objects from **mixsmsn** by Location Parameters*

Description

To sort an object returned from package **mixsmsn** by its location parameters

Usage

```
## S3 method for class 'Skew.normal'
sort(x, decreasing = FALSE, ...)
```

```
## S3 method for class 'Normal'
sort(x, decreasing = FALSE, ...)
```

```
## S3 method for class 'Skew.t'
sort(x, decreasing = FALSE, ...)
```

```
## S3 method for class 't'
sort(x, decreasing = FALSE, ...)
```

Arguments

x	Normal, Skew.normal, Skew.t object
decreasing	logical scalar. Should the sort the location parameter be increasing (FALSE, default) or decreasing (TRUE)?
...	additional parameters, currently not in use

Details

[smsn.mix](#) does **not** order the location parameter

Value

[sort.Normal](#) returns a Normal object.

[sort.Skew.normal](#) returns a Skew.normal object.

[sort.Skew.t](#) returns a Skew.t object.

See Also[sort](#)

step_fm	<i>Forward Selection of gh-parsimonious Model with Fixed Number of Components K</i>
---------	---

Description

To select the gh -parsimonious mixture model, i.e., with some g and/or h parameters equal to zero, conditionally on a fixed number of components K .

Usage

```
step_fm(
  object,
  test = c("BIC", "AIC"),
  direction = c("forward", "backward"),
  ...
)
```

Arguments

object	fmx object
test	character scalar, criterion to be used, either Akaike's information criterion AIC , or Bayesian information criterion BIC (default).
direction	character scalar, 'forward' (default) or 'backward'
...	additional parameters, currently not in use

Details

The algorithm starts with quantile least Mahalanobis distance estimates of either the full mixture of Tukey g -&- h distributions model, or a constrained model (i.e., some g and/or h parameters equal to zero according to the user input). Next, each of the non-zero g and/or h parameters is tested using the likelihood ratio test. If all tested g and/or h parameters are significantly different from zero at the level 0.05 the algorithm is stopped and the initial model is considered gh -parsimonious. Otherwise, the g or h parameter with the largest p-value is constrained to zero for the next iteration of the algorithm.

The algorithm iterates until only significantly-different-from-zero g and h parameters are retained, which corresponds to gh -parsimonious Tukey's g -&- h mixture model.

Value

[step_fm](#) returns an object of S3 class 'step_fm', which is a [list](#) of selected models (in reversed order) with attribute(s) 'direction' and 'test'.

See Also[step](#)

TukeyGH

*Tukey's g-&-h Distribution***Description**

Density, distribution function, quantile function and random generation for the Tukey's *g*-&-*h* distribution with location parameter *A*, scale parameter *B*, skewness *g* and kurtosis *h*.

Usage

```
dGH(x, A = 0, B = 1, g = 0, h = 0, log = FALSE, ...)
```

```
rGH(n, A = 0, B = 1, g = 0, h = 0)
```

```
qGH(p, A = 0, B = 1, g = 0, h = 0, lower.tail = TRUE, log.p = FALSE)
```

```
pGH(q, A = 0, B = 1, g = 0, h = 0, lower.tail = TRUE, log.p = FALSE, ...)
```

```
z2qGH(z, A = 0, B = 1, g = 0, h = 0)
```

```
qGH2z(q, q0 = (q - A)/B, A = 0, B = 1, ...)
```

Arguments

<code>x, q</code>	double vector , quantiles
<code>A</code>	double scalar, location parameter <i>A</i> , default $A = 0$ (as parameter mean of dnorm function)
<code>B</code>	double scalar, scale parameter $B > 0$, default $B = 1$ (as parameter sd of dnorm function)
<code>g</code>	double scalar, skewness parameter <i>g</i> , default $g = 0$ indicating no skewness
<code>h</code>	double scalar, kurtosis parameter $h \geq 0$, default $h = 0$ indicating no kurtosis
<code>log, log.p</code>	logical scalar, if TRUE, probabilities <i>p</i> are given as $\log(p)$.
<code>...</code>	other parameters of vuniroot2
<code>n</code>	integer scalar, number of observations
<code>p</code>	double vector , probabilities
<code>lower.tail</code>	logical scalar, if TRUE (default), probabilities are $Pr(X \leq x)$ otherwise, $Pr(X > x)$.
<code>z</code>	double vector , standard normal quantiles.
<code>q0</code>	double vector of $(q - A)/B$, for internal use to increase compute speed

Details

Argument A, B, g and h will be recycled to the maximum length of the four.

Value

dGH gives the density and accommodates **vector** arguments A, B, g and h. The quantiles x can be either **vector** or matrix. This function takes about 1/5 time of **dgh**.

pGH gives the distribution function, only taking scalar arguments and **vector** quantiles q. This function takes about 1/10 time of **pgh** and **pgh** functions.

qGH gives the quantile function, only taking scalar arguments and **vector** probabilities p. This function takes about 1/2 time of **qgh** and 1/10 time of **qgh** functions.

rGH generates random deviates, only taking scalar arguments.

z2qGH is the Tukey's *g*-&-*h* transformation. Note that `gk::z2gh` is only an **approximation** to Tukey's *g*-&-*h* transformation.

Unfortunately, **qGH2z**, the inverse of Tukey's *g*-&-*h* transformation, does not have a closed form and needs to be solved numerically.

See Also

[dgh](#) [dgh](#)

Examples

```
(x = c(NA_real_, rGH(n = 5L, g = .3, h = .1)))
dGH(x, g = c(0,.1,.2), h = c(.1,.1,.1))

p0 = seq.int(0, 1, by = .2)
(q0 = qGH(p0, g = .2, h = .1))
range(pGH(q0, g = .2, h = .1) - p0)

q = (-2):3; q[2L] = NA_real_; q
(p1 = pGH(q, g = .3, h = .1))
range(qGH(p1, g = .3, h = .1) - q, na.rm = TRUE)
(p2 = pGH(q, g = .2, h = 0))
range(qGH(p2, g = .2, h = 0) - q, na.rm = TRUE)

curve(dGH(x, g = .3, h = .1), from = -2.5, to = 3.5)
```

ud_allequal

Determine Nearly-Equal Elements

Description

Determine nearly-equal elements and extract non-nearly-equal elements in a **double vector**.

Usage

```
unique_allequal(x, ...)

duplicated_allequal(x, ...)
```

Arguments

x **double** vector
 ... additional parameters of [outer_allequal](#)

Value

[duplicated_allequal](#) returns a **logical vector** of the same length as the input vector, indicating whether each element is nearly-equal to any of the previous elements.

[unique_allequal](#) returns the non-nearly-equal elements in the input vector.

See Also

[outer_allequal](#) [duplicated.default](#) [unique.default](#)

Examples

```
x = c(.3, 1-.7, 0, .Machine$double.eps)
unique.default(x) # not desired
unique_allequal(x) # desired
```

 user_constraint

Formalize User-Specified Constraint of [fmx](#) Object

Description

Formalize user-specified constraint of [fmx](#) object

Usage

```
user_constraint(x, distname, K)
```

Arguments

x **character vector**, constraint(s) to be imposed. For example, for a two-component Tukey's *g*-&-*h* mixture, `c('g1', 'h2')` indicates $g_1 = h_2 = 0$ given $A_1 < A_2$, i.e., the *g*-parameter for the first component (with smaller location value) and the *h*-parameter for the second component (with larger mean value) are to be constrained as 0.

distname **character** scalar, name of distribution

K **integer** scalar, number of components

Value

`user_constraint` returns the indexes of internal parameters (only applicable to Tukey's *g*-&-*h* mixture distribution, yet) to be constrained, based on the type of distribution (`distname`), number of components (`K`) and a user-specified string (e.g., `c('g2', 'h1')`).

Examples

```
(d0 = fmx('GH', A = c(1,4), g = c(.2,.1), h = c(.05,.1), w = c(1,1)))
(c0 = fmx_constraint(d0))
stopifnot(identical(c0, user_constraint(distname = 'GH', K = 2L, x = character()))))

(d1 = fmx('GH', A = c(1,4), g = c(.2,0), h = c(0,.1), w = c(1,1)))
(c1 = fmx_constraint(d1))
stopifnot(identical(c1, user_constraint(distname = 'GH', K = 2L, x = c('g2', 'h1'))))

(d2 = fmx('GH', A = c(1,4), g = c(.2,0), h = c(.15,.1), w = c(1,1)))
(c2 = fmx_constraint(d2))
stopifnot(identical(c2, user_constraint(distname = 'GH', K = 2L, x = 'g2'))))
```

vcov.fmx

*Variance-Covariance of fmx Object***Description**

..

Usage

```
## S3 method for class 'fmx'
vcov(object, internal = FALSE, ...)
```

Arguments

<code>object</code>	<code>fmx</code> object
<code>internal</code>	logical scalar, either for the user-friendly parameters (<code>FALSE</code> , default) (e.g., mean, sd for normal mixture, and A,B,g,h for Tukey's <i>g</i> -and- <i>h</i> mixture), or for the internal/unconstrained parameters (<code>TRUE</code>).
<code>...</code>	place holder for S3 naming convention

Details

`vcov.fmx` returns the approximate asymptotic variance-covariance **matrix** of the user-friendly parameters via delta-method (`parm = 'user'`), or the asymptotic variance-covariance matrix of the internal/unconstrained parameters (`parm = 'internal'`). When the distribution has constraints on one or more parameters, `vcov.fmx` does not return the variance/covariance involving the constrained parameters.

Value

`vcov.fmx` returns a [matrix](#).

See Also

[vcov](#)

vuniroot2

Vectorised One Dimensional Root (Zero) Finding

Description

To solve a monotone function $y = f(x)$ for a given [vector](#) of y values.

Usage

```
vuniroot2(
  y,
  f,
  interval = stop("must provide a length-2 `interval`"),
  tol = .Machine$double.eps^0.25,
  maxiter = 1000L
)
```

Arguments

<code>y</code>	numeric vector of y values
<code>f</code>	monotone function $f(x)$ whose roots are to be solved
<code>interval</code>	length two numeric vector
<code>tol</code>	double scalar, desired accuracy (convergence tolerance),
<code>maxiter</code>	integer scalar, maximum number of iterations

Details

[vuniroot2](#), different from [vuniroot](#), does

- accept `NA_real_` as element(s) of y
- handle the case when the analytical root is at lower and/or upper
- return a root of `Inf` (if `abs(f(lower)) >= abs(f(upper))`) or `-Inf` (if `abs(f(lower)) < abs(f(upper))`), when the function value `f(lower)` and `f(upper)` are not of opposite sign.

Value

[vuniroot2](#) returns a [numeric vector](#) x as the solution of $y = f(x)$ with given vector y .

See Also[vuniroot](#)**Examples**

```

library(rstpm2)
lwr = rep(1, times = 9L); upr = rep(3, times = 9L)

# ?rstpm2::vuniroot does not accept NA \eqn{y}
tryCatch(vuniroot(function(x) x^2 - c(NA, 1:8), lower = lwr, upper = upr), error = identity)

# ?rstpm2::vuniroot not good when the analytic root is at `lower` or `upper`
f <- function(x) x^2 - 1:9
tryCatch(vuniroot(f, lower = lwr, upper = upr, extendInt = 'no'), warning = identity)
tryCatch(vuniroot(f, lower = lwr, upper = upr, extendInt = 'yes'), warning = identity)
tryCatch(vuniroot(f, lower = lwr, upper = upr, extendInt = 'downX'), error = identity)
tryCatch(vuniroot(f, lower = lwr, upper = upr, extendInt = 'upX'), warning = identity)

vuniroot2(c(NA, 1:9), f = function(x) x^2, interval = c(1, 3)) # all good

```

[,fmx,ANY,ANY,ANY-method

Subset of Components in [fmx](#) Object

Description

Taking subset of components in [fmx](#) object

Usage

```
## S4 method for signature 'fmx,ANY,ANY,ANY'
x[i]
```

Arguments

x [fmx](#) object

i [integer](#) or [logical vector](#), the row index(es) of **components** to be chosen, see [

Details

Note that using definitions as S3 method dispatch `[,fmx` won't work for [fmx](#) objects.

Value

An [fmx](#) object consisting of a subset of components. information about the observations (e.g. slots @data and @data.name), will be lost.

Examples

```
(d = fmx('norm', mean = c(1, 4, 7), w = c(1, 1, 1)))  
d[1:2]
```


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