

# SKAT Package

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## 1 Overview

SKAT package has functions to 1) test an association between SNP sets and continuous/binary phenotypes and 2) compute power/sample size for future sequence association studies.

## 2 Testing association between SNP sets and outcome phenotypes.

### 2.1 Example Dataset

SKAT package provides an example dataset (`SKAT.example`) that has a genotype matrix (`Z`) of 2000 individuals and 67 SNPs, a vector of continuous phenotypes (`y.c`), a vector of binary phenotypes (`y.b`) and a covariates matrix (`X`).

```
> library(SKAT)
> data(SKAT.example)
> names(SKAT.example)

[1] "Z"    "X"    "y.c"  "y.b"

> attach(SKAT.example)
```

To test an association, you first need to run `SKAT_Null_Model` function to get parameters and residuals from the null model of no association, and then to run `SKAT` to compute a p-value.

```
> # continuous trait
> obj<-SKAT_Null_Model(y.c ~ X, out_type="C")
> SKAT(Z, obj)$p.value

[1] 0.002877041

> # dichotomous trait
> obj<-SKAT_Null_Model(y.b ~ X, out_type="D")
> SKAT(Z, obj)$p.value
```

```
[1] 0.1401991
```

```
>
```

When the trait is binary and the sample size is small, SKAT can produce conservative results. We recently developed a small sample adjustment method that adjusts the asymptotic null distribution by estimating small sample moments. By default, SKAT (  $\geq$  ver 0.7) will conduct a small sample adjustment when the sample size  $< 2000$ . In the following code, we only use 200 samples to run SKAT.

```
> IDX<-c(1:100,1001:1100)
```

```
> # With-adjustment
```

```
> obj.s<-SKAT_Null_Model(y.b[IDX] ~ X[IDX,],out_type="D")
```

Sample size = 200, which is  $< 2000$ . The small sample adjustment is applied!

```
> SKAT(Z[IDX,], obj.s, kernel = "linear.weighted")$p.value
```

```
[1] 0.1339498
```

```
>
```

If you don't want to use the adjustment, please set Adjustment=FALSE when you run the SKAT\_Null\_Model function.

```
> # Without-adjustment
```

```
> obj.s<-SKAT_Null_Model(y.b[IDX] ~ X[IDX,],out_type="D", Adjustment=FALSE)
```

```
> SKAT(Z[IDX,], obj.s, kernel = "linear.weighted")$p.value
```

```
[1] 0.147093
```

## 2.2 Assign weights for each SNP

It is generally assumed that rarer variants have larger effect sizes. To incorporate it, the linear weighted kernel is formulated as  $ZWWZ'$ , where  $Z$  is a genotype matrix, and  $W = \text{diag}\{w_1, \dots, w_m\}$  is a weight matrix. In the previous examples, we have used the default  $\text{beta}(1,25)$  weight,  $w_i = \text{dbeta}(p_i, 1, 25)$ , where  $\text{dbeta}$  is the beta density function, and  $p_i$  is a minor allele frequency (MAF) of the  $i^{\text{th}}$  SNP. The beta weight with different parameter can be used by changing the weights.beta parameter. For example, if you want to use Madsen and Browning weight, use  $\text{weight.beta}=\text{c}(0.5,0.5)$ .

```
> SKAT(Z, obj, kernel = "linear.weighted", weights.beta=c(0.5,0.5))$p.value
```

```
[1] 0.4931639
```

If you want to use different types of weights, you should make your own weight vector and use it as weights parameter. For the logistic weight, we provide a function that generates it.

```

> # Shape of the logistic weight
>
> MAF<-1:1000/1000
> W<-Get_Logistic_Weights_MAF(MAF, par1=0.07, par2=150)
> par(mfrow=c(1,2))
> plot(MAF,W,xlab="MAF",ylab="Weights",type="l")
> plot(MAF[1:100],W[1:100],xlab="MAF",ylab="Weights",type="l")
> par(mfrow=c(1,2))
> # Use logistic weight
> weights<-Get_Logistic_Weights(Z, par1=0.07, par2=150)
> SKAT(Z, obj, kernel = "linear.weighted", weights=weights)$p.value

[1] 0.3293643

```

## 2.3 Unified Test

The test statistic of the unified test is

$$Q_\rho = (1 - \rho)Q_S + \rho Q_B,$$

where  $Q_S$  is a test statistic of SKAT, and  $Q_B$  is a score test statistic of weighted burden test. Thus,  $\rho = 0$  results in the original weighted linear kernel SKAT, and  $\rho = 1$  results in the weighted burden test. You can specify  $\rho$  value using the `r.corr` parameter (default: , `r.corr=0`).

```

> # Shape of the logistic weight
>
> #rho=0
> SKAT(Z, obj, r.corr=0)$p.value

[1] 0.1401991

> #rho=0.9
> SKAT(Z, obj, r.corr=0.9)$p.value

[1] 0.06031026

```

If `method="optimal.adj"`,  $\rho$  is selected from a grid of eight points  $\rho = (0, 0.1^2, 0.2^2, 0.3^2, 0.4^2, 0.5^2, 0.5, 1)$  to maximize the power. If you want to use the original implementation of SKAT-O, use `method="optimal"`. We recommend to use "optimal.adj", since it has a better type I error control in the tail area.

```

> #Optimal Test
> SKAT(Z, obj, method="optimal.adj")$p.value

[1] 0.1013505

>

```

## 2.4 Imputing missing genotypes.

If there are missing genotypes, SKAT automatically imputes them based on Hardy-Weinberg equilibrium. You can choose either “random” or “fixed” imputation (default=“fixed”). The “random” imputation generates  $\text{binomial}(2, p_i)$  random numbers to impute missing values, where  $p_i$  is the MAF of the  $i^{\text{th}}$  SNP calculated from non-missing genotypes, and the “fixed” imputation uses the mean genotype value,  $2p_i$ , to impute missing values.

```
> # Assign missing
> Z1<-Z
> Z1[1,1:3]<-NA
> # random imputation
> SKAT(Z1,obj,impute.method = "random")$p.value

[1] 0.1401991

> # fixed imputation
> SKAT(Z1,obj,impute.method = "fixed")$p.value

[1] 0.1401982
```

## 2.5 Resampling

SKAT package provides functions to conduct resampling methods to compute resampling p-values and to control family wise error rate. Two different resampling methods are implemented. “bootstrap” conducts the parametric bootstrap to resample residuals from  $H_0$  with considering covariates. When there is no covariate, “bootstrap” is equivalent to the permutation method. “perturbation” perturbs the residuals by multiplying mean zero and variance one normal random variables. The default method is “bootstrap”. From ver 0.7, we do not provide the “perturbation” method.

```
> # parametric bootstrap.
> obj<-SKAT_Null_Model(y.b ~ X, out_type="D", n.Resampling=5000,
+ type.Resampling="bootstrap")
> # SKAT p-value
> re<- SKAT(Z, obj, kernel = "linear.weighted")
> re$p.value          # SKAT p-value

[1] 0.1401991

> Get_Resampling_Pvalue(re)          # get resampling p-value

$p.value
[1] 0.1409718

$is_smaller
[1] FALSE
```

When there are many genes/SNP sets to test, resampling methods can be used to control family-wise error rate. You can find an example in the next section.

## 2.6 Plink Binary format files

SKAT package can read plink binary format files for genome-wide data analysis. To use plink files, plink bed, bim and fam files, and your own setid file that contains information of SNP sets are needed. Example files can be found on the SKAT webpage.

```
> # To run this code, first download and unzip example files
>
> #####
> #           Generate SSD file
>
> # Create the MW File
> File.Bed<-"./Example1.bed"
> File.Bim<-"./Example1.bim"
> File.Fam<-"./Example1.fam"
> File.SetID<-"./Example1.SetID"
> File.SSD<-"./Example1.SSD"
> File.Info<-"./Example1.SSD.info"
> # To use binary ped files, you have to generate SSD file first.
> # If you already have a SSD file, you do not need to call this function.
> Generate_SSD_SetID(File.Bed, File.Bim, File.Fam, File.SetID, File.SSD, File.Info)
```

```
Check duplicated SNPs in each SNP set
No duplicate
1000 Samples, 10 Sets, 984 Total SNPs
[1] "SSD and Info files are created!"
```

Now you can open SSD and Info file and run SKAT. After finishing using it, you must call close function to close SSD file.

```
> FAM<-Read_Plink_FAM(File.Fam, Is.binary=FALSE)
> y<-FAM$Phenotype
> # To use a SSD file, please open it first. After finishing using it, you must close it.
>
> SSD.INFO<-Open_SSD(File.SSD, File.Info)

1000 Samples, 10 Sets, 984 Total SNPs
Open the SSD file

> # Number of samples
> SSD.INFO$nSample

[1] 1000
```

```

> # Number of Sets
> SSD.INFO$nSets

[1] 10

> obj<-SKAT_Null_Model(y ~ 1, out_type="C")
> out<-SKAT.SSD.All(SSD.INFO, obj)
> out

$results
      SetID      P.value N.Marker.All N.Marker.Test
1  GENE_01 0.77747880          94          94
2  GENE_02 0.06245208          84          84
3  GENE_03 0.38416582         108         108
4  GENE_04 0.46179268         101         101
5  GENE_05 0.18548863         103         103
6  GENE_06 0.93255760          94          94
7  GENE_07 0.18897220         104         104
8  GENE_08 0.73081683          96          96
9  GENE_09 0.67366458         100         100
10 GENE_10 0.40310682         100         100

$P.value.Resampling
NULL

attr(,"class")
[1] "SKAT_SSD_ALL"

```

If you have more than one gene/SNP set to test an association, you should adjust multiple testing. It can be done either by conducting bonferroni correction or by estimating false discovery rate. However, if gene/SNP sets are correlated, these approaches would produce conservative results. Alternatively, you can directly control family wise error rate (FWER) using the resampling method. Example code is given in following.

```

> obj<-SKAT_Null_Model(y ~ 1, out_type="C", n.Resampling=1000, type.Resampling="bootstrap")
> out<-SKAT.SSD.All(SSD.INFO, obj)
> # No gene is significant with controlling FWER = 0.05
> Resampling_FWER(out,FWER=0.05)

$result
NULL

$n
[1] 0

$ID
NULL

```

```
> # 1 gene is significant with controlling FWER = 0.5
> Resampling_FWER(out,FWER=0.5)
```

```
$result
      SetID      P.value N.Marker.All N.Marker.Test
2 GENE_02 0.06245208           84           84
```

```
$n
[1] 1
```

```
$ID
[1] 2
```

If you want to test a single gene/SNP set, not all genes/SNP sets, you can use either “SKAT.SSD.OneSet” or “SKAT.SSD.OneSet\_SetIndex”. Or you can get a genotype matrix using “Get\_Genotypes\_SSD” function and then run SKAT. If you want to use different types of weights (ex. logistic weights), you should use this approach.

```
> obj<-SKAT_Null_Model(y ~ 1, out_type="C")
> # test the second gene
> id<-2
> SetID<-SSD.INFO$SetInfo$SetID[id]
> SKAT.SSD.OneSet(SSD.INFO,SetID, obj)$p.value

[1] 0.06245208

> SKAT.SSD.OneSet_SetIndex(SSD.INFO,id, obj)$p.value

[1] 0.06245208

> # test the second gene with the logistic weight.
> Z<-Get_Genotypes_SSD(SSD.INFO, id)
> weights = Get_Logistic_Weights(Z, par1=0.07, par2=150)
> SKAT(Z, obj, weights=weights)$p.value

[1] 0.7227001

>
```

After finishing, please close the SSD file.

```
> Close_SSD()
```

Close the opened SSD file: /private/var/folders/zs/nf\_6qpd12r1dm4v3y2y298fr0000gn/T/RtmpC4pi

### 3 Power/Sample Size calculation.

#### 3.1 Dataset

SKAT package provides a haplotype dataset (SKAT.haplotypes) which contains a haplotype matrix of 10,000 haplotypes over 200kb region (Haplotype), and a dataframe with informations of each SNP. These haplotypes were simulated using a calibrated coalescent model with mimicking linkage disequilibrium structure of European ancestry. If you don't have any haplotype information, please use this dataset to compute power/sample size.

```
> data(SKAT.haplotypes)
> names(SKAT.haplotypes)

[1] "Haplotype" "SNPInfo"

> attach(SKAT.haplotypes)
```

#### 3.2 Power/Sample Size calculation

SKAT package provides functions to compute the power/sample size for future sequence association studies. In the following example, we carried out sample size calculation using the haplotypes in SKAT.haplotypes with the following parameters.

1. Subregion length = 3k bp
2. Causal percent = 20%
3. Negative percent = 20%
4. For continuous traits,  $\beta = c|\log_{10}(MAF)|$  (BetaType = "Log") with  $\beta = 2$  at  $MAF = 10^{-4}$
5. For binary traits,  $\log(OR) = c|\log_{10}(MAF)|$  (OR.Type = "Log") with  $OR = 2$  at  $MAF = 10^{-4}$ , and 50% of samples are cases and 50% of samples are controls

```
> set.seed(500)
> out.c<-Power_Continuous(Haplotype,SNPInfo$CHROM_POS, SubRegion.Length=5000,
+ Causal.Percent= 20, N.Sim=10, MaxBeta=2,Negative.Percent=20)

[1] "10/10"

> out.b<-Power_Logistic(Haplotype,SNPInfo$CHROM_POS, SubRegion.Length=5000,
+ Causal.Percent= 20, N.Sim=10 ,MaxOR=7, Negative.Percent=20)

[1] "10/10"

> out.c
```



```

$Power
      0.01      0.001      1e-06
500  0.5601495 0.4507543 0.2745436
1000 0.6983510 0.6372979 0.4477310
1500 0.7393476 0.6978347 0.5840998
2000 0.7741144 0.7169529 0.6649380
2500 0.8041370 0.7386689 0.6938517
3000 0.8224103 0.7660432 0.6997755
3500 0.8349515 0.7896737 0.7015918
4000 0.8484832 0.8037123 0.7049269
4500 0.8647970 0.8109526 0.7122846
5000 0.8834324 0.8165985 0.7253563

$R.sq
[1] 0.0693529

attr("class")
[1] "SKAT_Power"

> out.b

$Power
      0.01      0.001      1e-06
500  0.3894872 0.2757429 0.1330505
1000 0.5888308 0.4573657 0.2436726
1500 0.7021843 0.5859396 0.3485361
2000 0.7763091 0.6650800 0.4668508
2500 0.8234240 0.7280271 0.5483447
3000 0.8516985 0.7775865 0.5943673
3500 0.8718116 0.8108489 0.6269605
4000 0.8899993 0.8317031 0.6603647
4500 0.9081573 0.8464714 0.6968862
5000 0.9262225 0.8594656 0.7324297

attr("class")
[1] "SKAT_Power"

> Get_RequiredSampleSize(out.c, Power=0.8)

$`alpha` = 1.00e-02`
[1] 2431.102

$`alpha` = 1.00e-03`
[1] 3867.782

$`alpha` = 1.00e-06`
[1] "> 5000"

```

```
> Get_RequiredSampleSize(out.b, Power=0.8)
```

```
$`alpha` = 1.00e-02`  
[1] 2251.417
```

```
$`alpha` = 1.00e-03`  
[1] 3336.919
```

```
$`alpha` = 1.00e-06`  
[1] "> 5000"
```

```
>
```

In this example, we used N.Sim=10 to get results quickly. When you do the power calculation, please increase it to more than 100. When BetaType = "Log" or OR.Type = "Log", the effect size of continuous trait and the log odds ratio of binary traits are  $c|\log_{10}(MAF)|$ , where  $c$  is determined by Max\_Beta or Max\_OR. For example,  $c = 2/4 = 0.5$  when the Max\_Beta = 2. In this case, a causal variant with MAF=0.01 has  $\beta = 1$ . For binary traits,  $c = \log(7)/4 = 0.486$  with MAX\_OR=7. And thus, a causal variant with MAF=0.01 has log OR = 0.972.

If you consider non-zero r.corr ( $\rho$ ) values to compute the power, Power\_Continuous\_R or Power\_Logistic\_R functions can be used instead. For example, r.corr=0 is SKAT and r.corr=1 is a burden test. Since they use slightly different method to compute the power, the powers from Power\_Continuous\_R and Power\_Logistic\_R can be slightly different from the powers from Power\_Continuous and Power\_Logistic although r.corr=0.

If you want to computer the power of SKAT-O by estimating the optimal r.corr, use r.corr=2. The estimated optimal r.corr is

$$r.corr = p_1^2(2p_2 - 1)^2,$$

where  $p_1$  is the proportion of nonzero  $\beta$ s, and  $p_2$  is the proportion of negative (or positive)  $\beta$ s among the non-zero  $\beta$ s.

```
> set.seed(500)  
> out.c<-Power_Continuous_R(Haplotype,SNPInfo$CHROM_POS, SubRegion.Length=5000,  
+ Causal.Percent= 20, N.Sim=10, MaxBeta=2,Negative.Percent=20, r.corr=2)  
  
[1] "10/10"
```

```
> out.c
```

```
$Power  
      0.01      0.001      1e-06  
500 0.5584048 0.4465557 0.2700370  
1000 0.6980094 0.6374870 0.4403217  
1500 0.7367947 0.6977547 0.5830013
```

```

2000 0.7707641 0.7148115 0.6664808
2500 0.8032711 0.7341910 0.6946357
3000 0.8253110 0.7606592 0.6998229
3500 0.8407660 0.7863270 0.7011542
4000 0.8569269 0.8038311 0.7035340
4500 0.8759197 0.8137950 0.7089662
5000 0.8968032 0.8214246 0.7192218

$R.sq
[1] 0.0693529

$r.corr
[1] 0.0144

attr("class")
[1] "SKAT_Power"

> Get_RequiredSampleSize(out.c, Power=0.8)

$`alpha = 1.00e-02`
[1] 2449.686

$`alpha = 1.00e-03`
[1] 3890.566

$`alpha = 1.00e-06`
[1] "> 5000"

>

```