

Knee Injuries - Proportional Odds Models

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At the beginning the knee dataset is loaded:

```
> library(catdata)
> data(knee)
> attach(knee)
```

First of all a simple χ^2 – test of independence between the therapy (Th) and the pain level (R4) is applied.

```
> suppressWarnings(chisq.test(Th,R4))
```

Pearson's Chi-squared test

```
data:  Th and R4
X-squared = 18.2, df = 4, p-value = 0.001128
```

In the following the variable Age is centered around 30 years and the quadratic variable Age2 is created.

```
> Age <- Age - 30
> Age2 <- Age^2
```

The response pain (R4) has to be an ordered factor, the covariates therapy (Th) and sex (Sex) need to be factors.

```
> R4 <- as.ordered(R4)
> Th <- as.factor(Th)
> Sex <- as.factor(Sex)
```

A proportional odds model can be fitted by the function "polr" from the "MASS" – library. Attention has to be paid to the algebraic signs of the coefficients. These are inverse to the usual interpretation in porportional odds models.

```
> library(MASS)
```

The first model only uses therapy as covariate, to achieve a proportional odds model the option "method" needs to use the logistic link function.

```
> polr1 <- polr(R4 ~ Th, method="logistic")
> summary(polr1)
```

```
Call:
polr(formula = R4 ~ Th, method = "logistic")
```

```
Coefficients:
      Value Std. Error t value
Th2 -0.893      0.328   -2.72
```

```
Intercepts:
      Value Std. Error t value
1|2 -1.466   0.285   -5.141
2|3 -0.286   0.255   -1.120
3|4  0.667   0.254    2.621
4|5  2.644   0.436    6.059
```

```
Residual Deviance: 373.20
AIC: 383.20
```

The corresponding odds-ratio can be retrieved by the following command (consider the inverse sign!):

```
> exp(-coef(polr1))

Th2
2.44
```

Now a model with the covariates therapy, sex and age is fitted.

```
> polr2 <- polr(R4 ~ Th + Sex + Age, method="logistic")
> summary(polr2)
```

```
Call:
polr(formula = R4 ~ Th + Sex + Age, method = "logistic")
```

```
Coefficients:
      Value Std. Error t value
Th2  -0.9438      0.336  -2.813
Sex1  0.0499      0.373   0.134
Age  -0.0159      0.017  -0.936
```

```
Intercepts:
      Value Std. Error t value
1|2 -1.453   0.409   -3.549
2|3 -0.269   0.394   -0.681
3|4  0.686   0.392    1.752
4|5  2.674   0.522    5.121
```

```
Residual Deviance: 372.25
AIC: 386.25
```

Odds-ratios for the second model:

```
> exp(-coef(polr2))
```

Th2	Sex1	Age
2.570	0.951	1.016

To get the Wald-statistic, the standard errors have to be extracted from the summary. Afterwards the Wald-statistic and the corresponding p-values are easily recieved.

```
> se <- summary(polr2)[1][[1]][1:3,2]
> (wald2 <- -coef(polr2)/se)
```

Th2	Sex1	Age
2.813	-0.134	0.936

P-values for the second model:

```
> 1-pchisq(wald2^2, df=1)
```

Th2	Sex1	Age
0.00491	0.89367	0.34921

Finally the quadratic age-effect is added to the previous model.

```
> polr3 <- update(polr2, ~. + Age2)
```

```
> summary(polr3)
```

Call:

```
polr(formula = R4 ~ Th + Sex + Age + Age2, method = "logistic")
```

Coefficients:

	Value	Std. Error	t value
Th2	-0.94452	0.33871	-2.7886
Sex1	-0.08295	0.37836	-0.2192
Age	0.00171	0.01804	0.0948
Age2	-0.00622	0.00209	-2.9766

Intercepts:

	Value	Std. Error	t value
1 2	-2.204	0.490	-4.497
2 3	-0.943	0.460	-2.050
3 4	0.065	0.446	0.145
4 5	2.082	0.557	3.738

Residual Deviance: 362.88

AIC: 378.88

Odds-ratios for the final model:

```
> exp(-coef(polr3))
```

Th2	Sex1	Age	Age2
2.572	1.086	0.998	1.006

Wald-statistic for the final model:

```
> se <- summary(polr3)[1][[1]][1:4,2]
> (wald3 <- -coef(polr3)/se)
```

Th2	Sex1	Age	Age2
2.7886	0.2192	-0.0948	2.9766

P-values for the final model:

```
> 1-pchisq(wald3^2, df=1)
```

Th2	Sex1	Age	Age2
0.00529	0.82647	0.92445	0.00291

As the proportional odds-model is the most popular model for ordinal data, there are several different ways to fit such models. Now the final model is additionally fitted with function "vglm" from the "VGAM"-library and with function "lrm" from the "rms"-library.

Model fitted with "vglm":

```
> library(VGAM)

> m.vglm <- vglm(R4 ~ Th + Sex + Age + Age2, family = cumulative (link="logit", parallel=T)
> summary(m.vglm)
```

Call:

```
vglm(formula = R4 ~ Th + Sex + Age + Age2, family = cumulative(link = "logit",
parallel = TRUE))
```

Pearson Residuals:

	Min	1Q	Median	3Q	Max
logit(P[Y<=1])	-2	-0.8	-0.2	0.9	2.9
logit(P[Y<=2])	-3	-0.5	0.3	0.9	1.8
logit(P[Y<=3])	-3	-0.2	0.3	0.4	1.4
logit(P[Y<=4])	-6	0.1	0.1	0.2	0.5

Coefficients:

	Value	Std. Error	t value
(Intercept):1	-2.204	0.460	-4.8
(Intercept):2	-0.943	0.424	-2.2
(Intercept):3	0.065	0.418	0.2
(Intercept):4	2.082	0.543	3.8
Th2	0.944	0.332	2.8
Sex1	0.083	0.357	0.2
Age	-0.002	0.018	-0.1
Age2	0.006	0.002	3.0

Number of linear predictors: 4

Names of linear predictors:

```
logit(P[Y<=1]), logit(P[Y<=2]), logit(P[Y<=3]), logit(P[Y<=4])
```

```
Dispersion Parameter for cumulative family: 1
```

```
Residual Deviance: 363 on 500 degrees of freedom
```

```
Log-likelihood: -181 on 500 degrees of freedom
```

```
Number of Iterations: 5
```

The resulting coefficients are very similar to the coefficients in the model above fitted with function "polr", but they have inverse signs. Therefore they can be interpreted in the usual way.

Model fitted with "lrm":

```
> library(rms)
```

```
> m.lrm <- lrm(R4 ~ Th + Sex + Age + Age2)
```

```
> m.lrm
```

Logistic Regression Model

```
lrm(formula = R4 ~ Th + Sex + Age + Age2)
```

Frequencies of Responses

```
1 2 3 4 5
36 34 25 26 6
```

		Model Likelihood Ratio Test	Discrimination Indexes	Rank Discrim. Indexes
Obs	127	LR chi2 17.87	R2 0.138	C 0.656
max deriv	3e-10	d.f. 4	g 0.811	Dxy 0.313
		Pr(> chi2) 0.0013	gr 2.249	gamma 0.317
			gp 0.183	tau-a 0.241
			Brier 0.210	

	Coef	S.E.	Wald Z	Pr(> Z)
y>=2	2.2040	0.4900	4.50	<0.0001
y>=3	0.9431	0.4600	2.05	0.0403
y>=4	-0.0647	0.4459	-0.15	0.8847
y>=5	-2.0819	0.5568	-3.74	0.0002
Th=2	-0.9445	0.3387	-2.79	0.0053
Sex=1	-0.0829	0.3784	-0.22	0.8265
Age	0.0017	0.0180	0.09	0.9245
Age2	-0.0062	0.0021	-2.98	0.0029

Again no big differences are found concerning the coefficients. Here the signs are the same as with the function "polr".